

Fun Logarithm Problems

1. Given $3 = k \cdot 2^r$ and $15 = k \cdot 4^r$, find r .
2. Given $x, y > 0$, $\log_y x + \log_x y = \frac{10}{3}$, and $xy = 144$, find $\frac{1}{2}(x + y)$.
3. Given $\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 x^2 + \log_8 x^3 + \log_{16} x^4 = 40$, find x .
4. Define $f(n) = \begin{cases} \log_8 n, & \text{if } \log_8 n \text{ is rational} \\ 0, & \text{otherwise} \end{cases}$. Find $\sum_{n=1}^{2010} f(n)$.
5. If $\log_2(\log_2(\log_2 x)) = 2$, then how many digits are in the base-ten representation for x ?
6. The set of all real numbers x for which $\log_{2010}(\log_{2009}(\log_{2008}(\log_{2007} x)))$ is defined is $\{x \mid x > c\}$. What is c ?
7. For all positive integers n , $f(n) = \log_{2010} n^2$. Let $N = f(5) + f(6) + f(67)$. Find N .
- 8.